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Persistence of opinion in the Sznajd consensus model: computer simulation

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Abstract. The density of never changed opinions during the Sznajd consensus-finding process decays with time t as $1/t^\theta$. We find $\theta \simeq 3/8$ for a chain, compatible with the exact Ising result of Derrida *et al.* In higher dimensions, however, the exponent differs from the Ising θ. With simultaneous updating of sublattices instead of the usual random sequential updating, the number of persistent opinions decays roughly exponentially. Some of the simulations used multi-spin coding.

PACS. 05.50.+q Lattice theory and statistics (Ising, Potts, etc.) – 89.65.-s Social systems

1 Introduction

In the Ising and Potts models, the persistent spins [1] are those which from the beginning of a (zero-temperature) Monte Carlo simulation have never been flipped. In the thermodynamic limit, their number decreases with time t asymptotically as $1/t^\theta$ with $\theta = 3/8$ exactly on the Ising chain [2], while higher dimensions were investigated numerically [3] giving $\theta \simeq 0.2$ on the square lattice. Also the more general Potts model was investigated [1–3]. A review was given in [4]. Now we simulate the analogous number in a d-dimensional Sznajd model of consensus-finding [5] with up to 49 million sites and $1 \leq d \leq 4$, for the case of just two possible opinions, the equivalent of spin 1/2 Ising sites.

In this Sznajd model (see [6] for a review) two opposing opinions are initially distributed randomly with equal probability over the L^d "people" of a hypercubic lattice. Then, each randomly selected pair of nearest neighbours convinces its $4d-2$ nearest neighbours of the pair opinion if the pair shares the same opinion; otherwise, the neighbour opinions are not affected. One time step means that on average every lattice site is selected once as the first member of the pair. (We will mention below the different results if this random sequential updating is replaced by simultaneous updating.) The Sznajd model is one of several recent consensus-finding models [7] and follows a long tradition of social studies using computer simulation and/or statistical physics [8]. If we wait sufficiently long for large systems, always a consensus is found: Everybody has the same opinion and the whole system has reached a fixed point.

Alternatively, independently of social interpretations, this model can be understood as a variant of the traditional kinetic Ising model: instead of a central site being influenced by its neighbourhood, the neighbourhood itself is updated according to the states of the central spins. In spite of this difference to the Ising model, the pictures of growing domains in the Sznajd model (published in [6]) look similar to Ising coarsening at low temperatures (except for single overturned Ising spins), and the cluster statistics scales in the same way [9] on the square lattice. Thus one might expect, wrongly as it will turn out, the same persistence exponents for Sznajd and Ising models.

2 Simulations

We check for the number $P(t)$ of "persistent" sites who have not yet changed their spins in this Sznajd consensus process. (All our sites are equivalent, in contrast to Schneider's modification [10] where some sites are initially selected as permanent opponents.) We find that usually a consensus is found before everybody had changed opinion; *i.e.* $P(\infty) > 0$. Thus the exponent θ has to be determined from intermediate times where $P(0) \gg P(t) \gg P(\infty)$, or from $P(t) - P(\infty)$. Figure 1 shows that this latter quantity has a complicated behaviour, and again only intermediate times are used to find θ. In one dimension, $P(\infty)$ is relatively small and the resulting systematic deviations disturb less.

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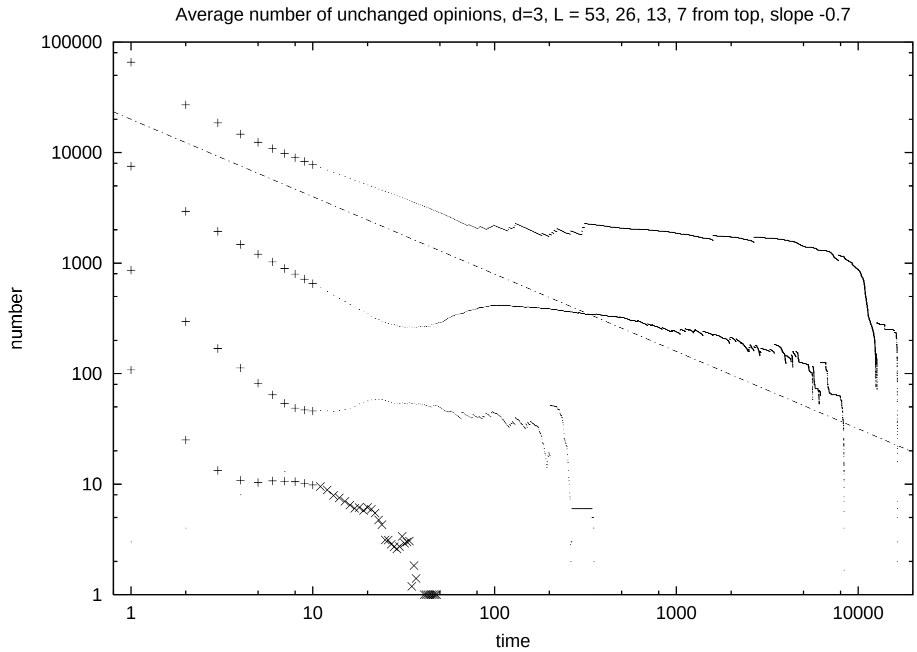

Fig. 1. Log-log plot of the number of persistent people, $P(t) - P(\infty)$, *versus* time, for one, 10^3, 10^3, 10^4 simple cubic lattices of size $L \times L \times L$ with $L = 53$, 26, 13, 7. Only the intermediate times before the plateau and final decay to zero are used to estimate the exponent $\theta \simeq 0.7$, indicated here by the straight line.

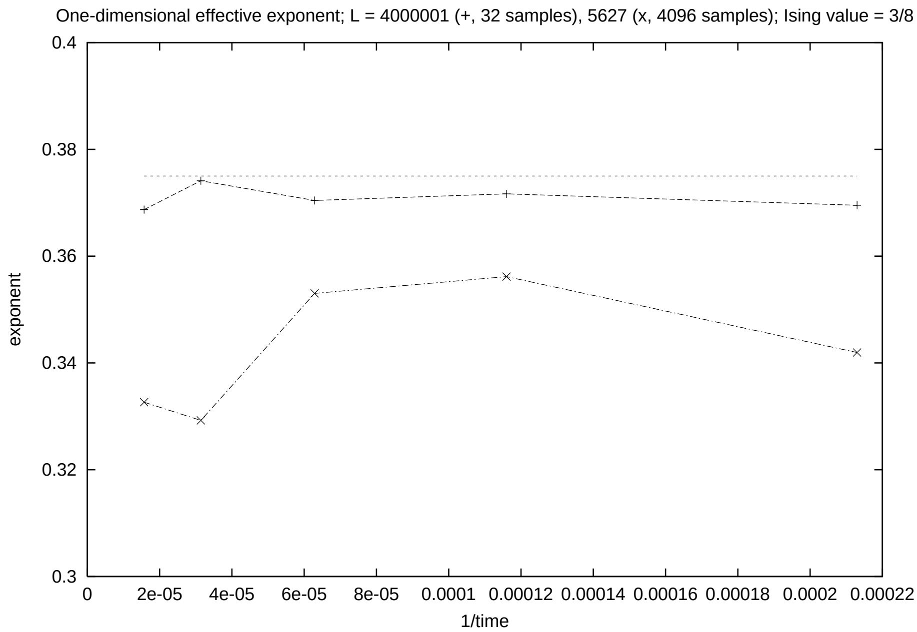

Fig. 2. Effective exponents in one dimension, approaching perhaps the Ising value 3/8 (horizontal line) for long times and large lattices.

Figures 2 to 4 show for $d = 1$, 2 and 3 the effective exponents θ analyzed by least square fits over five suitable time intervals $t_n < t < t_{n+1}$ with $t_{n+1} = 2t_n$. For $d > 1$, only the times until the first of the (typically 1000) samples reached a consensus were used and averaged over. We conclude that $\theta \simeq 3/8$ in one dimension, 0.5 in two, and the same or somewhat higher in three dimensions. Figure 5 shows that four dimensions is difficult to analyze though maybe $\theta \simeq 0.9$. Our one-dimensional estimate is compatible with the Ising value 3/8, but for higher dimensions our $\theta(d)$ goes up while the Ising $\theta(d)$ went down for increasing d.

We also speeded up the simulations by storing 32 or 64 sites (belonging to 32 or 64 different samples) in each computer word, using single-bit handling [11] known for Ising models as multi-spin coding. The random

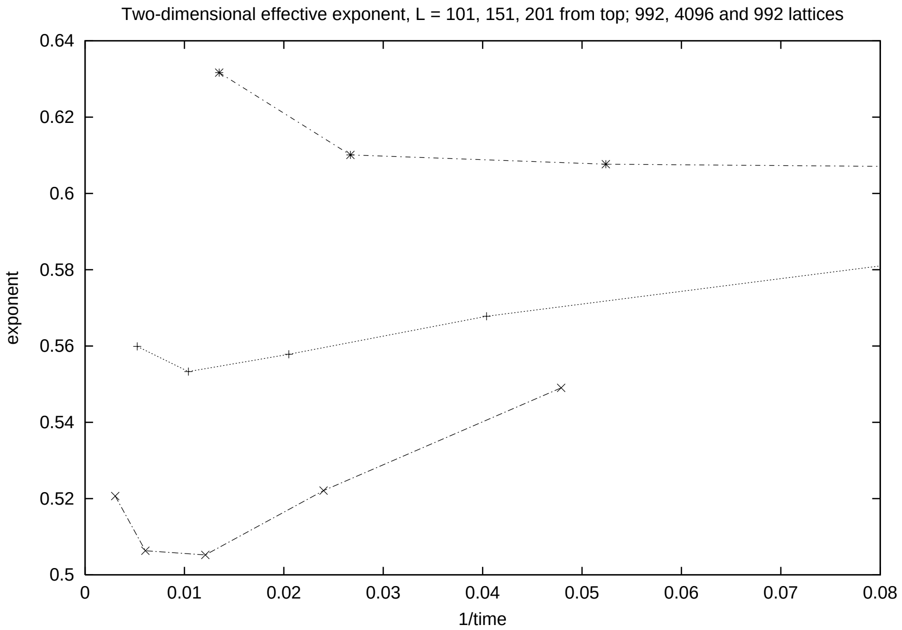

Fig. 3. As Figure 2, but for square lattices, using intermediate times only, before any final consensus was found.

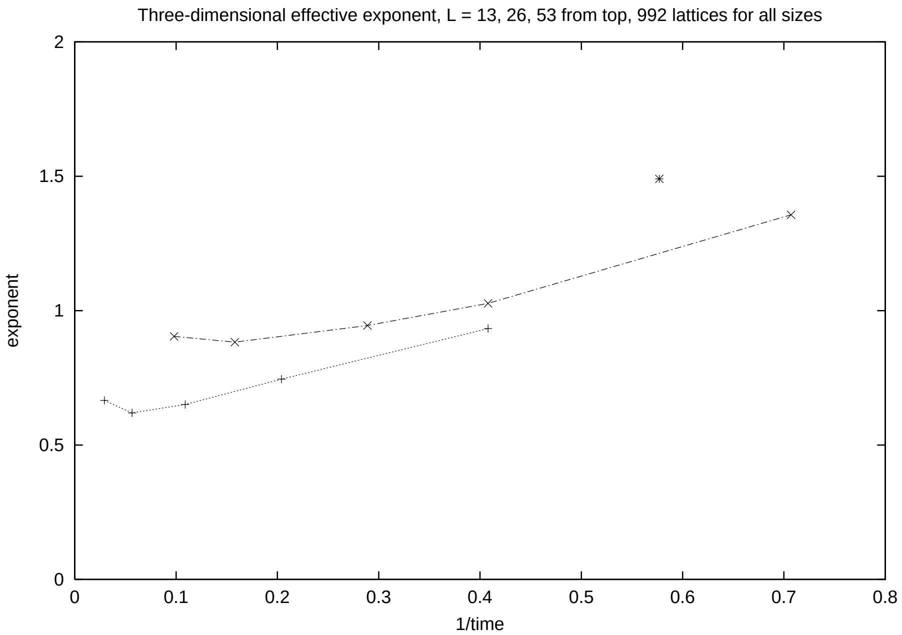

Fig. 4. As Figure 3 but for simple cubic lattices.

selection of neighbour pairs was the same for all 32 or 64 samples. The C program is available from PMCO, the Fortran program from DS. The C program uses periodic boundary conditions, the Fortran program has free boundaries. Both give the same results for the short times analyzed here and differ in the final number of unchanged sites after a fixed point was reached.

If during one time step all sites are updated simultaneously, with frustrated sites not changing their opinion, then no consensus is found [12]. (Frustrated are those sites which simultaneously are convinced to different opinions by different neighbour pairs.) This frustration can be avoided by dividing the lattice into sublattices, such that no sites within one sublattice can influence each other directly; we divided our lattices such that the distances between sites belonging to the same sublattice are at least five lattice constants. (For nearest-neighbour Ising models, the two sublattices of a chess-board suffice on the square lattice, while we used 25 inter-penetrating sublattices for the square Sznajd model.) With this simultaneous updating of sublattices, frustration is avoided, a consensus is always found, but $P(t)$ no longer decays as a power law, Figure 6: criticality seems lost. Also, this version no longer shows the phase transition of the usual square Sznajd

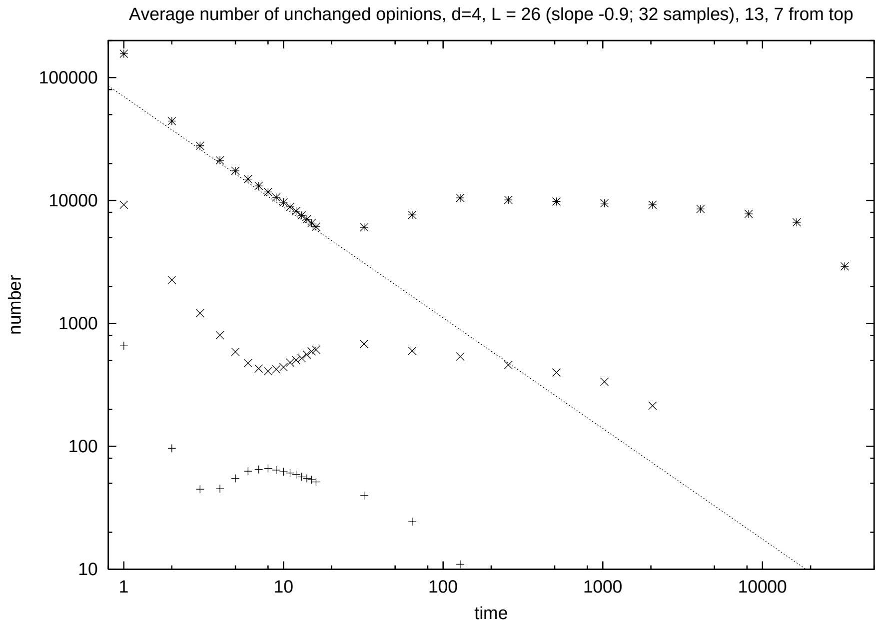

Fig. 5. Log-log plot of $P(t) - P(\infty)$ for hypercubic lattices in four dimensions averaged over 10^3 samples (32 samples only for $L = 26$).

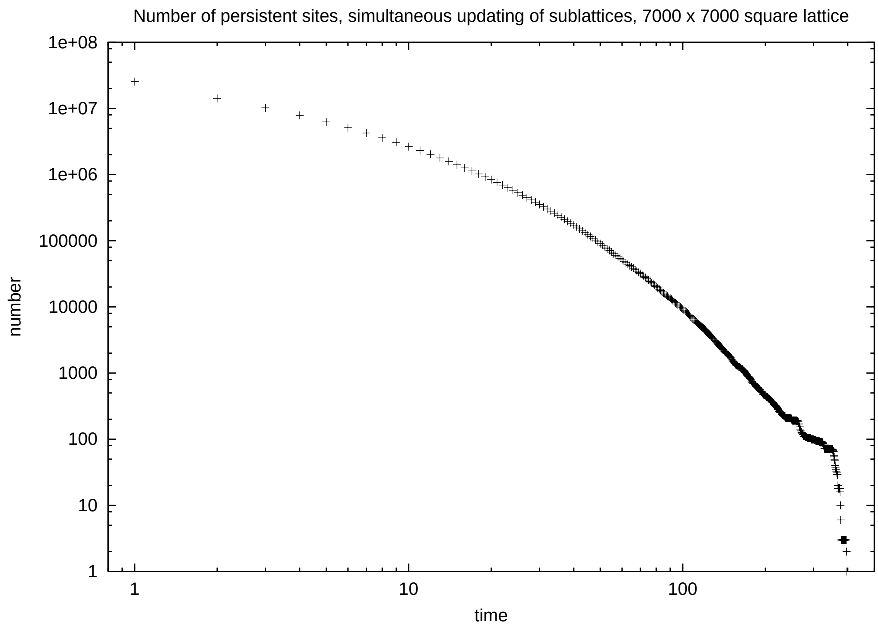

Fig. 6. Log-log plot of $P(t)$ for 7000×7000 square lattice with unfrustrated simultaneous updating of sublattices.

model, when the ratio of the fractions of the two initial opinions is varied away from unity. Thus, the simultaneous updating is not merely a possible acceleration of the dynamic process. Also the correlations between spins behave differently, being affected by the simultaneous updating of spins far apart from each other.

3 Discussion

How can we explain these results? An anonymous referee gave the following argument why in one dimension the Sznajd and Ising exponents for persistence are the same: If one draws a picture of a configuration, it becomes evident that the dynamics of the domain walls (separating the up and down domains) perform simple random walks and annihilate upon contact, as in the one-dimensional Ising model. There are two minor differences with the Ising model: (i) the rates of diffusion of the walkers are slightly different from the Ising model, which can be taken care of by rescaling the time and (ii) unlike the Ising model, the fully antiferromagnetic configuration is an attractor of the dynamics in the Sznajd model. The dynamics stops if the system gets into this configuration. However, this is

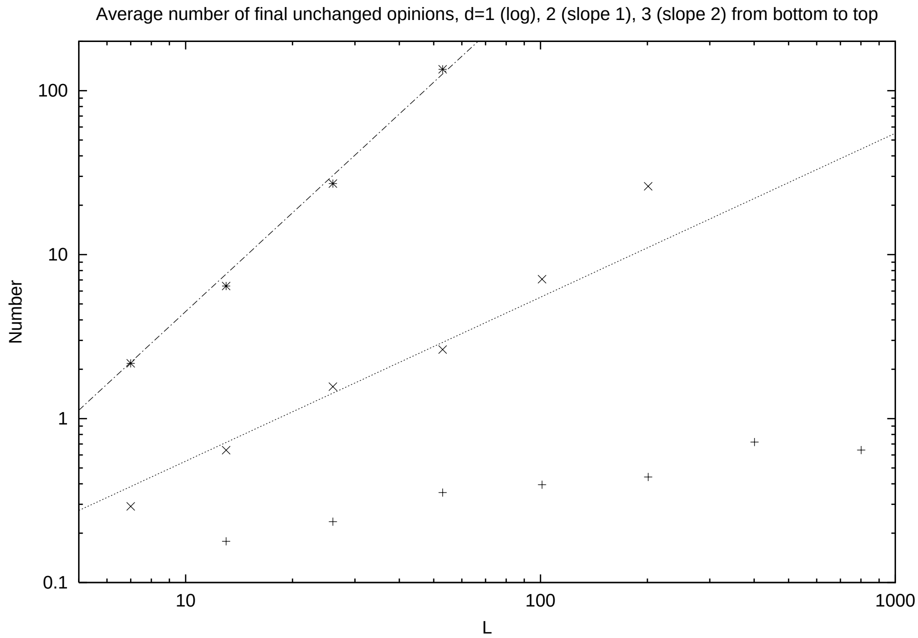

Fig. 7. Variation with system size of the final number of never changed opinions, in one to three dimensions, using free boundary conditions. Different size dependencies were seen with periodic boundary conditions (not shown).

an unstable configuration, so a large enough system never reaches there if it starts from a random initial configuration. So, at early times, when there are blocks of antiferromagnetic domains, the dynamics will be different from the Ising model but very soon the system will get rid of these antiferromagnetic blocks and then a typical configuration will consist of only up and down domains separated by diffusing domain walls, as in the Ising model. This argument is not rigorous since a single down spin in a long line of up spins constitutes two freely moving domain walls for the Ising case but can only vanish in the Sznajd case.

Our final number of never changed spins, which disturbed the analysis at the very end of our plots, Figure 1, is not a blocking effect as may happen in Ising models. Instead, we have here two competing processes leading to a fixed point at which the simulations stop: Process A is the consensus finding in the standard Sznajd model; after a sufficiently long time no spin could be flipped by the dynamic rules; the system reaches a globally absorbing state, for instance all spins parallel to each other. Process B is the monotonic decay of the number of never changed opinions; if that number reaches zero our particular simulation here also stops since this number will stay at zero even if the spins still flip. The persistence exponent, according to which the number of never flipped spins decay as a power law, corresponds only to process B, still during the transient regime. The purpose of our work is to study only this process B, not the final regime of process A, where the system becomes dead. If process A finishes before process B is finished, some opinions remain unchanged forever and lead to our difficulties. The data in Figure 7, based on free boundary conditions, suggest that the number of never changed opinions in a lattice of L^d sites varies roughly as L^{d-1}, and thus the fraction of such opinions vanishes as $1/L$. (The exponent zero in one dimension may indicate a logarithmic increase.) However, these details depend on boundary conditions; our analysis for the exponents thus concentrated on the earlier times where none of the many samples yet had finished its process A.

In summary, the Sznajd model is Ising-like in one dimension but not in higher dimensions for the persistence exponent θ.

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